QED Effective Actions in Inhomogeneous Backgrounds: Summing the Derivative Expansion

Gerald V. Dunne

Department of Physics, University of Connecticut, Storrs, CT 06269-3046, USA

Abstract. The QED effective action encodes nonlinear interactions due to quantum vacuum polarization effects. While much is known for the special case of electrons in a constant electromagnetic field (the Euler-Heisenberg case), much less is known for inhomogeneous backgrounds. Such backgrounds are more relevant to experimental situations. One way to treat inhomogeneous backgrounds is the "derivative expansion", in which one formally expands around the soluble constant-field case. In this talk I use some recent exactly soluble inhomogeneous backgrounds to perform precision tests on the derivative expansion, to learn in what sense it converges or diverges. A closely related question is to find the exponential correction to Schwinger's pair-production formula for a constant electric field, when the electric background is inhomogeneous.

This talk is concerned with the one-loop QED effective action [1,2]:

$$S[A] = -\frac{i}{2}\log\det\left(\not{D}^{\,2} + m^2\right) \tag{1}$$

Here $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative, and so $S[A]$ is a functional of the classical background field $A_\mu(x)$. The effective action is the generating functional for one-fermion-loop Green's functions, which describe the nonlinear QED effects to this order. Ideally, one would like to know $S[A]$ for *any* field $A_\mu(x)$, but this is not feasible. However, for the special case where the field strength tensor, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, is uniform, the effective action can be evaluated in closed form [3–5]:

$$S = \frac{e^2}{2}(\vec{E}^2 - \vec{B}^2) + \frac{2\alpha^2}{45m^4}\int d^4x\, \left[(\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E}\cdot\vec{B})^2\right] + \dots \tag{2}$$

Here the first term is the familiar classical Maxwell action, while the next term gives the leading quantum correction, which is quartic in the field strengths. The fine structure constant $\alpha = \frac{e^2}{4\pi}$ in these units. All higher terms in the expansion (2) are known explicitly [3–5]. It can also be shown that when the background field is a constant electric field of strength E, the effective action has an exponentially small imaginary part

$$\mathrm{Im}S = \frac{e^2E^2}{8\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left[-\frac{m^2\pi n}{eE}\right] \tag{3}$$

This imaginary part of S gives the pair-production rate for electron-positron production from vacuum in a strong uniform electric field [3,5,6]. Note also that it is manifestly nonperturbative in the coupling constant e, in contrast to the perturbative series expansion in (2).

For a general inhomogeneous background field we need some sort of approximation to determine the effective action. One such approximation is the derivative expansion [7–10], in which one assumes that $F_{\mu\nu}$ is "slowly varying", so that one can make the following formal expansion

$$S = S^{(0)}[F] + S^{(2)}[F,(\partial F)^2] + \dots \tag{4}$$

where $S^{(2n)}$ contains $2n$ derivatives of $F_{\mu\nu}$. Clearly, this is a rather formal and ill-defined expansion. In this talk, I ask two specific questions about this expansion:

1. Does the derivative expansion converge?
2. How is the nonperturbative imaginary piece in (3) modified?

These two questions are in fact intimately related.

First, let us study the convergence properties of the constant-field Euler-Heisenberg effective action (2). When the uniform background is purely magnetic, the entire series (we ignore the Maxwell term) in (2) is

$$S = -\frac{2m^4}{\pi^2}\left(\frac{eB}{m^2}\right)^4\sum_{n=0}^{\infty}\frac{2^{2n}\mathcal{B}_{2n+4}}{(2n+4)(2n+3)(2n+2)}\left(\frac{eB}{m^2}\right)^{2n} \tag{5}$$

This is a "low energy" effective action in the sense that we assume that the characteristic cyclotron energy of the magnetic background, $\frac{eB}{m}$, is much less than the electron rest energy, m. The expansion coefficients in (5) involve Bernoulli numbers $\mathcal{B}_{2n}$, the first few of which are : $\frac{1}{6}$, $-\frac{1}{30}$, $\frac{1}{42}$, $-\frac{1}{30}$, $\frac{5}{66}$, ... The Bernoulli numbers [11] alternate in sign and grow factorially in magnitude. In fact, for large n,

$$\frac{-2^{2n}\mathcal{B}_{2n+4}}{(2n+4)(2n+3)(2n+2)} \sim (-1)^n\frac{1}{8\pi^4}\frac{\Gamma(2n+2)}{\pi^{2n}}\left(1+\frac{1}{2^{2n+4}}+\frac{1}{3^{2n+4}}+\dots\right) \tag{6}$$

Thus, the Euler-Heisenberg effective action is itself a divergent series! This is not a bad thing; perturbation theory is typically divergent. In many examples (the anharmonic oscillator, the Stark effect, ϕ^3, ϕ^4, QED, ...) perturbation theory is known to be divergent [12]. Typically, the expansion coefficients diverge as

$$|a_n| \sim \beta^n\,\Gamma(\gamma\, n+\delta)\,[1+...] \tag{7}$$

at large orders n, and usually the parameter $\gamma = 1$ or 2.

Even though a series may be divergent, we can still extract important and meaningful information from it. One such approach is known as Borel summation [13,14].

To motivate this approach, consider the following expression for the (clearly divergent) alternating series

$$f(g) = \sum_{n=0}^{\infty} (-1)^n \, n! \, g^n \sim \frac{1}{g} \int_0^{\infty} ds \, \frac{e^{-s/g}}{1+s} \tag{8}$$

Read from right to left, (8) says that the asymptotic expansion for small g of the integral on the RHS (which converges for all g positive) yields the series on the LHS. Read from left to right, (8) involves a formal interchange of summation and integration (a step whose validity must be examined carefully in a given case). We call the integral on the RHS the Borel sum of the divergent series on the LHS. If we were to try this for a non-alternating series, we might write analogously

$$f(-g) = \sum_{n=0}^{\infty} n! \, g^n \sim \frac{1}{g} \int_0^{\infty} ds \, \frac{e^{-s/g}}{1-s} \tag{9}$$

This is problematic, as there is a pole at $s = 1$ on the integration contour. This pole must be resolved, and in doing so, an imaginary part arises. Roughly speaking, this imaginary part is of the form $Im f(-g) = \frac{\pi}{g} \exp[-\frac{1}{g}]$, which is non-perturbative in g. Similar arguments apply if the expansion coefficients grow like $|a_n| \sim \beta^n \, \Gamma(\gamma \, n + \delta)$, rather than just $n!$ as in the above example. Then $f(g) = \sum_n a_n g^n$ behaves like

$$f(g) \sim \frac{1}{\gamma} \int_0^{\infty} \frac{ds}{s} \left(\frac{1}{1+s}\right) \left(\frac{s}{\beta g}\right)^{\delta/\gamma} \exp[-\left(\frac{s}{\beta g}\right)^{1/\gamma}] \tag{10}$$

$$Im \, f(-g) \sim \frac{\pi}{\gamma} \left(\frac{1}{\beta g}\right)^{\delta/\gamma} \exp[-\left(\frac{1}{\beta g}\right)^{1/\gamma}] \tag{11}$$

I caution that these Borel representations are somewhat formal, as we are assuming that there are no additional poles or cuts in the complex g plane to prevent the straightforward analytic continuation on which these expressions are based [13,14]. In this talk our attitude is exploratory; we shall use these formulae cautiously, with cross-checks, but defer questions such as uniqueness to more rigorous studies.

It is an instructive exercise to apply this Borel technology to the alternating divergent Euler-Heisenberg effective action (5) [15–17]. Combining the asymptotic form (6) of the coefficients with the Borel formula (10) we obtain

$$S = -\frac{e^2 B^2}{8\pi^2} \int_0^{\infty} \frac{ds}{s^2} \left(\coth s - \frac{1}{s} - \frac{s}{3}\right) \exp[-\frac{m^2 s}{eB}] \tag{12}$$

where we have used: $\coth s - 1/s = 2s/\pi^2 \sum_{k=1}^{\infty} 1/(k^2 + s^2/\pi^2)$, and $\sum_{k=1}^{\infty} 1/k^2 = \pi^2/6$. But this is precisely Schwinger's proper-time integral representation [5,3,4] of the effective action, which we see can be viewed as the Borel sum of the (divergent) Euler-Heisenberg perturbation series (5). If the constant background field is electric

instead of magnetic, the only change perturbatively is to replace B^2 in (5) by $-E^2$ (because the Lorentz invariant combination is $E^2 - B^2$). Thus the alternating divergent series (5) becomes a non-alternating divergent series. Applying the Borel formula (11), together with the asymptotic growth rate in (6), we find an imaginary part in complete agreement with Schwinger's result (3). So, in this case, our use of the Borel formulae (10-11) is consistent. Further, we see that the Euler-Heisenberg series (5) *had to be* divergent. If it were not, then there would be no essential difference between the magnetic and electric cases, and we would not find any imaginary part for the electric case; so we would miss the genuine physical effect of vacuum instability. This is somewhat reminiscent of Dyson's physical argument [18] that QED perturbation theory should be non-analytic as a series in $\alpha = \frac{e^2}{4\pi}$, since α negative is unstable.

Unfortunately, the pair production rate derived from (3) is extremely small. Indeed, the critical electric field, $E_c = \frac{m^2c^3}{e\hbar} \sim 10^{16}\, V/cm$, where the exponent is of order 1, is well beyond laboratory static fields. However, using intense laser fields, it has recently become possible to probe this critical regime [19]. These involve inhomogeneous fields, and so it becomes important to ask how the Euler-Heisenberg analysis is modified by a field inhomogeneity. Thus we return to our two questions concerning the convergence properties of the derivative expansion (4) and how this might modify the imaginary part (3).

A first guess is that one should look at the high orders of the derivative expansion to see if it is diverging. Unfortunately this is impossible for two fundamental reasons. First, the derivative expansion is not actually a series. This is because as one goes to higher orders, there is a rapid proliferation of new terms that do not have a counterpart at previous orders [8–10]. Second, it is extremely difficult to calculate any of these terms at very high order, so we could not expect anyway to observe any asymptotic behaviour of the expansion coefficients.

However, we can avoid both these problems at once by considering some special exactly solvable cases [15]. In these cases, the background inhomogeneity can be characterized by a single scale parameter, so that the derivative expansion becomes a true series, in inverse powers of this scale parameter. And since these cases are soluble, we have access to *all orders* of the derivative expansion, so that we can probe the high order behaviour. So, while we sacrifice generality by concentrating on specific backgrounds, we gain the ability to perform precise analytic tests of the derivative expansion.

To be specific, we will use the fact that for the following backgrounds, the effective action has been computed in closed-form [20,21].

$$\vec{B}(x) = \vec{B}\,\mathrm{sech}^2(\frac{x}{\lambda}) \qquad or \qquad \vec{E}(t) = \vec{E}\,\mathrm{sech}^2(\frac{t}{\tau}) \tag{13}$$

It has of course been known for many years that the corresponding Dirac equations are soluble [22–24]. But it is still non-trivial to perform explicitly the necessary traces so as to express the effective action (1) as a series or as an integral representation, in terms of a single integral, just as in the Euler-Heisenberg case (5) or

(12). For the inhomogeneous magnetic background, $\vec{B}(x) = \vec{B}\,\mathrm{sech}^2(\frac{x}{\lambda})$, in (13), the exact effective action is [20]

$$S = -\frac{m^4}{8\pi^{3/2}} \sum_{j=0}^{\infty} \sum_{k=1}^{\infty} \frac{1}{(m\lambda)^{2j}} \left(\frac{2eB}{m^2}\right)^{2k} \frac{\Gamma(2k+j)\Gamma(2k+j-2)\mathcal{B}_{2k+2j}}{j!(2k)!\Gamma(2k+j+\frac{1}{2})} \tag{14}$$

Several comments are in order concerning this result. First, note that the series expansion in (14) is a *double sum*, with derivative expansion parameter, $\frac{1}{m\lambda}$, and perturbative expansion parameter, $\frac{eB}{m^2}$. Second, the expansion coefficients are known to all orders, and are relatively simple numbers, just involving the Bernoulli numbers and factorial factors. Third, it has been checked in [20] that the first few terms of this derivative expansion are in agreement with explicit field theoretic calculations, specialized to this particular background. Fourth, a simple integral representation for (14) can be found in [20].

Given the explicit series representation in (14), we can check that the series is divergent, but Borel summable (in the sense of our earlier discussion), in the magnetic case. This can be done in several ways. One can either fix the order k of the perturbative expansion in (14) and show that the remaining sum is Borel summable, or one can fix the order j of the derivative expansion in (14) and show that the remaining sum is Borel summable. Or, one can sum explicitly the k sum, for each j, as an integral of a hypergeometric function, and show that for various values of $\frac{eB}{m^2}$, the remaining derivative expansion is divergent but Borel summable. These arguments do not prove rigorously that the double series is Borel summable, but give a strong numerical indication that this is the case.

The case of the inhomogeneous electric field, $\vec{E}(t) = \vec{E}\,\mathrm{sech}^2(\frac{t}{\tau})$, in (13) can also be solved explicitly [21]. A short-cut to the answer is to note that we can simply make the replacements, $B^2 \to -E^2$, and $\lambda^2 \to -\tau^2$, in the magnetic case result (14). In particular this has the consequence that the alternating divergent series of the magnetic case becomes a non-alternating divergent series, just as was found in the Euler-Heisenberg constant-field case. For example, fixing the order j of the derivative expansion, the expansion coefficients behave for large k (with j fixed) as

$$a_k^{(j)} = (-1)^{j+k} \frac{\Gamma(2k+j)\Gamma(2k+j+2)\mathcal{B}_{2k+2j+2}}{\Gamma(2k+3)\Gamma(2k+j+\frac{5}{2})} \sim 2\frac{\Gamma(2k+3j-\frac{1}{2})}{(2\pi)^{2j+2k+2}} \tag{15}$$

Note that these coefficients are non-alternating and grow factorially with $2k$, as in the form of (7). Applying the Borel dispersion formula (11) gives

$$\mathrm{Im}S^{(j)} \sim \frac{m^4}{8\pi^3}\left(\frac{eE}{m^2}\right)^{5/2} \exp\left[-\frac{m^2\pi}{eE}\right] \frac{1}{j!} \left(\frac{m^4\pi}{4\tau^2 e^3 E^3}\right)^j \tag{16}$$

Remarkably, this form can be resummed in j, yielding a leading exponential form

$$\mathrm{Im}S \sim \frac{m^4}{8\pi^3}\left(\frac{eE}{m^2}\right)^{5/2} \exp\left[-\frac{m^2\pi}{eE}\left\{1 - \frac{1}{4}\left(\frac{m}{eE\tau}\right)^2\right\}\right] \tag{17}$$

We recognize the first term in the exponent as the familiar Schwinger exponent from (3), and so the second term may be viewed as the leading *exponential* correction to the constant-field answer (3). This is what we set out to find, and we see that it arose through the divergence of the derivative expansion. I stress that this exponential correction is not accessible from low orders of the derivative expansion, such as those studied in [8–10].

But the situation is even more interesting than this result (17) suggests. For example, we could instead have considered doing the Borel resummation for the j summations, at each fixed k. Then for large j, the coefficients behave as [15]

$$a_j^{(k)} = (-1)^{j+k} \frac{\Gamma(j+2k)\Gamma(j+2k-2)\mathcal{B}_{2k+2j}}{\Gamma(j+1)\Gamma(j+2k+\frac{1}{2})} \sim 2^{9/2-2k} \frac{\Gamma(2j+4k-\frac{5}{2})}{(2\pi)^{2j+2k}} \tag{18}$$

which once again are non-alternating and factorially growing. Applying the Borel dispersion formula (11) gives

$$\mathrm{Im}S^{(k)} \sim \frac{m^{3/2}}{4\pi^3\tau^{5/2}} \frac{(2\pi eE\tau^2)^{2k}}{(2k)!}\, e^{-2\pi m\tau} \tag{19}$$

Once again we see that this leading form can be resummed, leading to

$$\mathrm{Im}S \sim \frac{m^{3/2}}{8\pi^3\tau^{5/2}} \exp\left[-2\pi m\tau\left(1-\frac{eE\tau}{m}\right)\right] \tag{20}$$

But this leading exponential form of the imaginary part is different from that obtained in (17), and moreover, it is different from the constant-field case (3). So, what is going on? The answer is that there are two competing leading exponential behaviours buried in the double sum (14), and the question of which one dominates depends crucially on the relative magnitudes of the two expansion parameters. These two expansion parameters are the derivative expansion parameter, $\frac{1}{m\tau}$, and the perturbative expansion parameter, $\frac{eE}{m^2}$. Another important parameter is their *ratio*, since this sets the scale of the corresponding gauge field:

$$\frac{A(t)}{m} = \frac{eE\tau\tanh(t/\tau)}{m} \sim \frac{eE\tau}{m} = \frac{eE/m^2}{1/(m\tau)} \tag{21}$$

Thus, we can define a "non-perturbative" regime, in which $\frac{eE\tau}{m} \gg 1$. Then $m\tau \gg \frac{m^2}{eE}$, so that the dominant exponential factor is $\exp[-\frac{m^2}{eE}] \gg \exp[-2\pi m\tau]$. In this regime, the leading imaginary contribution to the effective action is given by the expression (17), and we note that it is indeed non-perturbative in form, and the correction in the exponent is in terms of the small parameter $\frac{m}{eE\tau} \ll 1$. Alternatively, we can define a "perturbative" regime, in which $\frac{eE\tau}{m} \ll 1$. Then $m\tau \ll \frac{m^2}{eE}$, so that the dominant exponential factor is $\exp[-2\pi m\tau] \gg \exp[-\frac{m^2}{eE}]$. In this regime, the leading imaginary contribution to the effective action is given by

the expression (20), and we note that this is in fact perturbative in nature (despite its exponential form).

To understand these two regimes more carefully, we can use the WKB approach developed by Brézin and Itzykson [25], who studied the case of a sinusoidal electric field: $E(t) = E\cos(\omega t)$. This case is not exactly soluble, but a WKB expression for the imaginary part of the effective action is:

$$ImS \sim \int d^3k\, \exp[-\pi\Omega] \tag{22}$$

where $\Omega = \frac{2i}{\pi}\int_{\rm tp}\sqrt{m^2+k_\perp^2+(k_z-eA_z(t))^2}$. If we apply this same WKB analysis to the (exactly soluble) case $E(t) = E\,\mathrm{sech}^2(t/\tau)$ in (13) we find that $\Omega = \tau(\sqrt{m^2+k_\perp^2+(eE\tau+k_z)^2}+\sqrt{m^2+k_\perp^2+(eE\tau-k_z)^2}-2eE\tau)$. Moreover, this system is known to be WKB-exact [26,24]. Then when the momentum integrals in (22) are done, we obtain precisely the leading results (17) or (20), depending on whether we are in the "non-perturbative" $\frac{eE\tau}{m}\gg 1$, or "perturbative" $\frac{eE\tau}{m}\ll 1$ regime [15]. This serves as a useful cross-check of our Borel analysis.

To conclude, there exist two inhomogeneous backgrounds (one magnetic and one electric) for which the effective action (1) can be evaluated exactly, in closed-form, thereby generalizing the well-known Euler-Heisenberg constant-field case. In this talk I have used these cases to probe the convergence properties of the derivative expansion. We learn that the derivative expansion is divergent. In the magnetic case it appears to be Borel summable, and in the electric case the direct application of Borel dispersion relations leads to imaginary parts that are in complete agreement with an independent WKB analysis. Indeed, we can argue à la Dyson that the derivative expansion must be divergent, otherwise there would be no exponential correction to the imaginary part of S for an inhomogeneous electric field. There are two competing leading exponentials, and in the non-perturbative regime, where $\frac{eE\tau}{m}\gg 1$, we find an exponential correction (17) to Schwinger's formula (3). In the perturbative regime, where $\frac{eE\tau}{m}\ll 1$, a different exponential factor dominates.

REFERENCES

1. W. Dittrich and M. Reuter, *Effective Lagrangians in Quantum Electrodynamics*, (Springer, Berlin, 1985).
2. W. Greiner and J. Reinhardt, *Quantum Electrodynamics*, (Springer, Berlin, 1992).
3. W. Heisenberg and H. Euler, "Folgerungen aus der Diracschen Theorie des Positrons", Z. Phys. **98**, 714 (1936).
4. V. Weisskopf, "Uber die Elektrodynamik des Vakuums auf Grund der Quantentheorie des Elektrons", Kong. Dans. Vid. Selsk. Math-fys. Medd. XIV No. 6 (1936), reprinted in *Quantum Electrodynamics*, J. Schwinger (Ed.) (Dover, New York, 1958).
5. J.Schwinger, "On Gauge Invariance and Vacuum Polarization", Phys. Rev. **82**, 664 (1951).

6. F. Sauter,"Über das Verhalten eines Elektrons im homogenen elektrischen Feld nach der relativistischen Theorie Diracs", Z. Phys. **69**, 742 (1931).
7. I. J. R. Aitchison and C. M. Fraser, "Derivative Expansions of Fermion Determinants: Anomaly Induced Vertices, Goldstone-Wilczek Currents and Skyrme Terms", Phys. Rev. D **31**, 2605 (1985).
8. D. Cangemi, E. D'Hoker and G. Dunne, "Derivative Expansion of the Effective Action and Vacuum Instability for QED in $2+1$ Dimensions", Phys. Rev. D **51**, R2513 (1995).
9. V.P.Gusynin and I.A.Shovkovy, "Derivative Expansion for the One-Loop Effective Lagrangian in QED", Can. J. Phys. **74**, 282 (1996).
10. D. Fliegner, P. Haberl, M.G. Schmidt, and C. Schubert, "The Higher Derivative Expansion of the Effective Action by the String Inspired Method (Part 2)", Ann. Phys.(NY) **264**, 51 (1998).
11. The Bernoulli numbers $\mathcal{B}_{2n}$ are defined by $x/(e^x-1) = \sum_{n=0}^{\infty} \mathcal{B}_n x^n/n!$.
12. For an excellent review, see: J. C. Le Guillou and J. Zinn-Justin (Eds.), *Large-Order Behaviour of Perturbation Theory*, (North Holland, Amsterdam, 1990).
13. G. Hardy, *Divergent Series* (Oxford Univ. Press, 1949).
14. C. M. Bender and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers* (McGraw-Hill, New York, 1978).
15. G. Dunne and T. Hall, "Borel summation of the derivative expansion and effective actions", Phys. Rev. D **60**, 065002 (1999).
16. A. Zhitnitsky, "Is an Effective Lagrangian a Convergent Series?", Phys. Rev. D **54**, 5148 (1996).
17. G. Dunne and C. Schubert, "Two-loop Euler-Heisenberg QED pair-production rate", Nucl. Phys. B **564**, 591 (2000).
18. F. J. Dyson, "Divergence of Perturbation Theory in Quantum Electrodynamics", Phys. Rev. **85**, 631 (1952).
19. D. Burke *et al*, "Positron Production in Multiphoton Light-by-Light Scattering", Phys. Rev. Lett. **79**, 1626 (1997); A. Melissinos, "The Spontaneous Breakdown of the Vacuum", hep-ph/9805507.
20. G. Dunne, T. Hall, "An Exact QED_{3+1} Effective Action", Phys. Lett. B **419**, 322 (1998); D.Cangemi, E.D'Hoker, G.Dunne, "Effective Energy for QED_{2+1} with Semi-localized Static Magnetic Field: A Solvable Model", Phys. Rev. D **52**, R3163 (1995).
21. G. Dunne and T. Hall, "QED effective action in time dependent electric backgrounds", Phys. Rev. D **58**, 105022 (1998).
22. F. Sauter, "Zum 'Kleinschen Paradoxon'", Z. Phys. **73**, 547 (1932); this includes a related solvable Dirac equation for a spatially inhomogeneous electric background.
23. See, e.g., N. Narozhnyĭ and A. Nikishov, "The Simplest Processes in a Pair-producing Electric Field", Sov. J. Nucl. Phys. **11**, 596 (1970), and refs. therein.
24. A. B. Balantekin, J. E. Seger and S. H. Fricke, "Dynamical Effects in Pair Production by Electric Fields", Int. J. Mod. Phys. A **6**, 695 (1991).
25. E. Brézin and C. Itzykson, "Pair production in Vacuum by an Alternating Field", Phys. Rev. D **2**, 1191 (1970); E. Brézin, Thèse (Paris, 1970).
26. A. Comtet, A. Bandrauk and D. Campbell, "Exactness of Semiclassical Bound State Energies for Supersymmetric Quantum Mechanics", Phys. Lett. **150 B**, 159 (1985).